

The Magic Square of Order 3 Complete

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(Using High School Algebra)

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The object of the exercise is to arrange the numbers 1 to 9 in a square array so that when you are done the three numbers in each row, each column and both diagonals sum the same (called the Magic Sum). All the numbers from 1 to 9 sum 45. If each row is to be the same sum, then the row sum must be 15. We do not know what the numbers are yet, so in Figure 1 nine different letters are placed. We can see:

one diagonal	$a + e + i = 15$(1)
one column	$b + e + h = 15$(2)
another diagonal	$c + e + g = 15$(3)

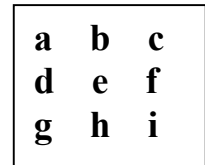


Figure 1.

Adding these equations, we get:

$$a + b + c + 3e + g + h + i = 45 \quad \text{.....(4)}$$

But, one row $a + b + c = 15 \quad \text{.....(5)}$

And another row $g + h + i = 15 \quad \text{.....(6)}$

Both Equations (6) may be subtracted to obtain:

$$3e = 15$$

or,

$$e = 5$$

FIVE MUST GO IN THE CENTER

Now, $a + b + c = 15 \quad \text{.....(5)}$

and, $a + d + g = 15 \quad \text{.....(7)}$

adding, we get

$$2a + b + c + d + g = 30. \quad \therefore$$

other diagonal $c + 5 + g = 15$

and by subtracting

$$2a + b + d = 20 \quad \text{.....5)}$$

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which means “ $b + d$ ” must be an even number. This is because:

- $2a$ is even. Twice any number is an even number,
- 20 is an even number
- and the sum of even numbers is always even.

Now, “ $b + d$ ” is even, in turn, means b & d have the same parity, That is either b & d are both even, or they are both odd. Similarly one could prove that b and f have the same parity, and also d and h .

If b, d, f, h are even then a, c, g, i (the corners) are odd, and vice-versa because there are only the two sets $1, 3, 7, 9$ and $2, 4, 6, 8$ available to be placed. If the odd numbers occupy the corners, then all the numbers must be odd because the sum of the sum 15 . This cannot be, so::

THE EVEN NUMBERS MUST GO IN THE CORNERS.

After placing a 5 in the center, there are four choices of an even number for a . Then i is determined and there are only two even numbers to place, So, for each choice of a there will be 2 choices of c . In other words, there will be eight possible magic squares out of nine factorial = 362880 ways of placing the numbers. The third-order magic square is considered the simplest one.

2	9	4
7	5	3
6	1	8

You can make the other seven of these by using this method, and then discover which are rotations, which are considered as reflections and which are both.