The Magic Square of Order 3 Complete

by **John Robert Hendricks**

(Using High School Algebra)

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The object of the exercise is to arrange the numbers 1 to 9 in a square array so that when you are done the three numbers in each row, each column and both diagonals sum the same (called the Magic Sum). All the numbers from 1 to 9 sum 45. If each row is to be the same sum, then the row sum must be 15. We do not know what the numbers are yet, so in Figure 1 nine different letters are placed. We can see:

one diagonal	a + e + i = 15	…..(1)
one column	b + e + h = 15	..…(2)
another diagonal	c + e + g = 15	…..(3)

a	**b**	**c**
d	**e**	**f**
g	**h**	**i**

Figure 1.

Adding these equations, we get:

a + b + c + 3e + g + h + i = 45 …..(4)

But, one row	a + b + c = 15 …..(5)
And another row	g + h + i = 15 …..(6)

Both Equations (6) may be subtracted to obtain:

3e =15

or,

e = 5

FIVE MUST GO IN THE CENTER

Now,	a + b + c = 15 …..(5)
and,	a + d + g = 15 …..(7)
adding, we get	
	2a+b+c+d+g =30. :
other diagonal	c + 5 + g =15
and by subtracting	
	2a+b+d = 20 …..5)

which means "b + d" must be an even number. This is because:

- 2a is even. Twice any number is an even number,
- 20 is an even number
- and the sum of even numbers is always even.

Now, "b + d" is even, in turn, means b & d have the same parity, That is either b & d are both even, or they are both odd. Similarly one could prove that b and f have the same parity, and also d and h.

If b, d, f, h are even then a, c, g, i (the corners) are odd, and vice-versa because there are only the two sets 1, 3, 7, 9 and 2, 4, 6, 8 available to be placed. If the odd numbers occupy the corners, then all the numbers must be odd because the sum of the sum 15. This cannot be, so::

THE EVEN NUMBERS MUST GO IN THE CORNERS.

After placing a 5 in the center, there are four choices of an even number for a. Then i is determined and there are only two even numbers to place, So, for each choice of a there will be 2 choices of c. In other words, there will be eight possible magic squares out of nine factorial = 362880 ways of placing the numbers. The third-order magic square is considered the simplest one.

2	**9**	**4**
7	**5**	**3**
6	**1**	**8**

You can make the other seven of these by using this method, and then discover which are rotations, which are considered as reflections and which are both.