Making Magic Squares by Patterns

Introduction

I show streamlining of the old system of components to more meaningful mathematics where the number of digits in the magic square, cube. or hypercube are the dimension; and, where the order of the hypercube is in the radix of the number system. This approach is meant for anyone,, especially children and those people teaching them.. In many cases "other number systems" are taught at schools, and then abandoned forevermore,

Counting

When you learned to count, you learned partially with your fingers as a guide. Since you have ten fingers. ten would be the basis, At school, you kept track of numbers by having them in columns like thousands, hundreds, tens and the number of units. So, **1,234** was understood as one thousand, two hundred and thirty four.

You don't have to use ten and its multiples as the basis for counting. Instead, you could have thought that one hand contains four fingers and a thumb. Using the four fingers alone, you could have set up your method of keeping track numbers in terms of numbers of 16s, numbers of fours and numbers of units and so **123** would mean:

: - one sixteen instead of one hundred,

- two fours, instead of two tens, and
- three units..

The number system that most people now use is based on multiples of ten and is called the decimal number system from rhe Latin word *decem* = ten.. Some ancient civilizations used other systems, The number system using four as a basis is called the quaternary number system from rhea Latin word *quotuor* = four. You do not see it around too much. You do see counting by 12s though in a dozen eggs. A dozen dozen eggs, is called a gross. But, in order to do magic squares, cubes and hypercubes of order four by pattern we might find it easier to use the quaternary system of numbers and then convert it to the decimal system afterwards. A conversion table is shown so that may go from one number system to the other.

Conversion Table

Decimal Number	Quaternary Number	Decimal Number	Quaternary Number	Decimal Number	Quaternary Number	Decimal Number	Quaternary Number	Decimal Number	Quaternary Number
0	0	4	10	8	20	12	30	16	100
1	1	5	11	9	21	13	31	17	101
2	2	6	12	10	22	14	32	18	102
3	3	7	13	11	23	15	33	19	103

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Table 1. Decimal Numbers and Quaternary numbers side-by-side.

The Patterns

Some magic squares, especially the inlaid ones, are made from patterns. Figure 3 shows a pattern for a magic square of order 4.

A PATTERN

α	β	γ	δ
γ	δ	α	β
δ	γ	β	α
β	α	δ	γ

Figure 2. Greek Square

Sometimes Greek letters are used in mathematics because we run out of symbols quickly,

We have: **alpha** α
beta β
gamma γ
delta δ

αa	βc	γb	δd
γd	δb	αc	βa
δc	γa	βd	αb
βb	αd	δa	γc

Figure 3

a	c	b	d
d	b	c	a
c	a	d	b
b	d	a	c

Figure 4. A Latin Square.

Please notice in Figures 2 & 3 that there is only one alpha (α) in each row, each column and each diagonal and that the same is true for beta, gamma and delta. Figure 2 is called a Greek Square because it has Greek letters in it. We use the Latin letters: a, b, c, d when we write in the English language. The thing to notice here is that there are only two configurations of symbols possible -- one as in Figure 2 and the other as in Figure 4. In Figures 2 and 4 you will find the letter a, for e**xample, and the letter a** in different positions in the square which makes Figure 3 possible. to combine every combination and permutation.

The Magic Square

αa	**βc**	**γb**	**δd**
γd	**δb**	**αc**	**βa**
δc	**γa**	**βd**	**αb**
βb	**αd**	**δm**	**γc**

Figure3. A Magic Square

Figure 3, is already a magic square because you can take any line of symbols and assume they are digits with the Greek symbol as the fours digit and the Latin letter as the units digit and all lines sum the same.

Decimal numbers have ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. But, in the number system base 4 there are only 4 digits 0, 1, 2 and 3. You have 4 choices for alpha and a, then choices for beta and b. etc. This means that you can make 24x24 = 576 magic squares with this pattern.

Let $\alpha = a = 0$
$\beta = b = 1$
$\gamma = c = 2$ then, put the numbers into the square of Figure 5 by
$\delta = d = 3$ replacing the letters. and you have a magic square.

Step 1. Find a pattern	Step 2. Assign Numbers	Step 3, Change to decimal
αa βc γb δd γd δb αc βa δc γa βd αb βb αd δa γc	**00 12 21 33** **23 31 02 10** **32 20 13 01** **11 03 30 22**	**00 06 09 15** **11 13 02 04** **14 08 07 01** **05 03 12 10**
figure 3.(again)	**Figure 5. Base 4**	**Figure 6. Base 10.**

The final step by this method, is to add one to each element and bring Figure 6 in line with the practice of using the numbers 1 to 36 which are the requirements a will also find that many of the magic squares will turn out to by rotations and1 reflections of others. \ Figure 7 turns out to represent a group of squares.

1	**7**	**10**	**16**
12	**14**	**3**	**5**
``**15**	**9**	**8**	**2**
6	**4**	**13**	**11**

Figure 7. Magic Square