Magic Squares of Doubled Order

by

John R. Hendricks

11	18	25	2	9
10	12	19	21	3
4	6	13	20	22
23	5	7	14	16
17	24	1	8	15

A magic square of order 5

Begin with any magic square you know. Then build one of twice the order which is four times the size.

36	18	100	52	59	34	2	75	43	86
60	37	19	96	53	3	71	44	87	35
54	56	38	20	97	72	45	88	31	4
98	55	57	39	16	41	89	32	5	73
17	99	51	58	40	90	33	1	74	42
67	49	26	83	15	65	8	76	24	92
48	30	82	14	66	91	64	7	80	23
29	81	13	70	47	22	95	63	6	79
85	12	69	46	28	78	21	94	62	10
11	68	50	27	84	9	77	25	93	61

A magic square of order ten.

NOTE

This booklet is not for sale. It becomes a further appendix to "Inlaid Magic Squares and Cubes." It is distributed free-of-charge with the remaining supply of the first edition.

70	71	52	58	64	39	8	2	46	95
53	59	65	66	72	22	16	40	84	28
61	67	73	54	60	10	29	98	42	11
74	55	56	62	68	43	87	31	5	24
57	63	69	75	51	76	50	19	13	32
82	38	44	25	1	26	100	94	88	7
49	30	6	12	93	18	37	81	80	99
36	17	23	79	35	85	4	48	92	86
3	9	90	41	47	97	91	15	34	78
20	96	27	33	14	89	83	77	21	45

Figure

308 - 151 St. Andrews St.,
Victoria, B.C., V8V 2M9,
CANADA

15 October 1999.

Minor Announcement

Discovered during the spring of 1999, was a new method of making magic squares of order 2k. An example shown top left is a tenth-order magic square which sums 505 in rows, columns and diagonals. In the second quadrant, you will find inlaid a 5th-order magic square which sums 315. Inlaid squares and various methods abound, so this simply adds another method into the system.

MAJOR ANNOUNCEMENT

The technique mentioned above, can be extended to three and four-dimensional space and higher. A magic tesseract of order six, with an inlaid magic tesseract of order three has been made. It contains the numbers from 1 to 1296 and sums 3,891 in the required 872 different ways. This is the world's first magic tesseract of order six.

The inlaid magic tesseract of order three sums 1,824, in the required 116 different ways. This becomes the world's first inlaid magic tesseract.

The new method for magic squares will be taken into account in the upcoming Second Edition of Inlaid Magic Squares and Cubes, which is unscheduled at the moment.

John R. Hendricks

John R. Hendricks

Appendix

Magic Squares of Doubled Order

For: Magic Squares of Order 2k, k=3, or more.

Shown in the book *Magic Squares to Tesseracts by Computer*, is a method of construction of magic squares of order 4k+2. Here is a different method which can double the size, or order based on similar but different reasoning.

Both systems use auxiliary, or supplementary, or special component squares, which are fully reflected into all quadrants. Both use a table.

Strachey's technique tells how to place the elements in the special component. My technique uses a Latin square to do that. Strachey's table is order vs. elements. Mine is octant vs. element and yields completely different results.

I include this in the book *Inlaid Magic Squares and Cubes* partly because some of the resulting squares are inlaid, but mostly because this is the first book going to Second Edition. This method is programmable, whereas Strachey's method is difficult to program on pocket calculators.

This technique uses six simple steps to make a magic square of doubled order from a given magic square.

Step #1
Make a Table.

This is the key step. The remainder follows and is easy. "m" is the order of the magic square you have been given. "Q" are the four quadrants of the magic square of order 2m which you would like to make. Quadrants here has a slightly different definition[see page A25.]

Q	0	1	2	3	...	m-1	Sums
1	c_{11}	c_{21}	c_{31}	c_{41}	...	c_{m1}	S_1
2	c_{12}	c_{22}	c_{32}	c_{42}	...	c_{m2}	S_2
3	c_{13}	c_{23}	c_{33}	c_{43}	...	c_{m3}	S_1
4	c_{14}	c_{24}	c_{34}	c_{44}	...	c_{m4}	S_2
Sums:	$6m^2$	$6m^2$	$6m^2$	$6m^2$	...	$6m^2$	$6m^3$

Table 1. A generalized table.

In Table 1, you will notice the four quadrants in the left-hand vertical column. Along the top of the table are the digits 0, 1, ...,m-1 found in a Latin square, or similar square. You will notice only two possible sums for each row in the right-hand column and the sum of those sums is $6m^3$. Along the bottom of the table, each sum must be $6m^2$. In the central part of the table are values c_{ij} which are the numbers:

$$0,\ m^2,\ 2m^2 \text{ and } 3m^2$$

in some order, or other and there are m sets of these numbers.

In order to make your table, you must make sure that there is one of each of: 0, m^2, $2m^2$, and $3m^2$ in every column. It does not matter what the rows are as long as they sum the two sums and that those four sums sum $6m^3$.

Different combinations and permutations of 0, m^2, $2m^2$ and $3m^2$ are simply tried by trial-and-error so that the border conditions are fulfilled. Once one table is made, you can most likely see how another different one can be made, and so on.

For example, if you had been given the third-order magic square, a very small table is required. But, there are several possibilities.

Q	0	1	2	Sum
1	$2m^2$	$3m^2$	m^2	$6m^2$
2	0	m^2	$2m^2$	$3m^2$
3	m^2	$2m^2$	$3m^2$	$6m^2$
4	$3m^2$	0	0	$3m^2$
S	$6m^2$	$6m^2$	$6m^2$	$18m^2$

Table 1a.

Q	0	1	2	Sum
1	$3m^2$	$2m^2$	m^2	$6m^2$
2	m^2	0	$2m^2$	$3m^2$
3	0	$3m^2$	$3m^2$	$6m^2$
4	$2m^2$	m^2	0	$3m^2$
S	$6m^2$	$6m^2$	$6m^2$	$18m^2$

Table 1b.

Q	0	1	2	Sum
1	$3m^2$	m^2	m^2	$5m^2$
2	m^2	0	$3m^2$	$4m^2$
3	0	$3m^2$	$2m^2$	$5m^2$
4	$2m^2$	$2m^2$	0	$4m^2$
S	$6m^2$	$6m^2$	$6m^2$	$18m^2$

Table 1c.

Q	0	1	2	Sum
1	$3m^2$	$2m^2$	0	$5m^2$
2	m^2	0	$3m^2$	$4m^2$
3	0	$3m^2$	$2m^2$	$5m^2$
4	$2m^2$	m^2	m^2	$4m^2$
S	$6m^2$	$6m^2$	$6m^2$	$18m^2$

Table 1d.

In these tables, you can interchange columns 1 and 2 leaving Q, 0, and sum fixed. You will note the requirements are met. As the tables increase in size with the order, so do the number of possibilities.

Rows of the same sum can be interchanged.

Arbitrarily, the column headed by a zero is designated to be the column which will control the diagonals of the larger magic square which will be made. In order to have the diagonals in the magic square of order 2m summing correctly, you will find that:

$$c_{11}+c_{13} = c_{12}+c_{14} = 3m^2$$

in Table 1 and you can see the same is true for the tables above. It is easier to use 0, m^2, $2m^2$, $3m^2$ as algebraic numbers in making the table than to evaluate it at this time.

The sums S_1 and S_2 may sometimes be equal, as is the case in even-ordered initial squares.

Step #2.
Fully Reflected Basic Latin Square

0	1	2	3	...	m-2	m-1
m-1	0	1	2	...	m-3	m-2
m-2	m-1	0	1	...	m-4	m-3
...	...	...	...	...	...	...
2	3	4	5	...	0	1
1	2	3	4	...	m-1	0

Figure 1.A basic Latin Square.

The Latin Square shown in Figure 1 has zeros running down one diagonal. Latin squares require one of each element in rows and columns and do not include diagonals. So, in this diagram the other digits lie in parallel diagonals. Unlike Strachey's method, we employ a Latin square, as above. But, it must be mirror-reflected and lake-reflected (both) into the four quadrants of a square of order 2m. For example:

	0	1	2	2	1	0	
Q2	2	0	1	1	0	2	Q1
	1	2	0	0	2	1	
	1	2	0	0	2	1	
Q3	2	0	1	1	0	2	Q4
	0	1	2	2	1	0	

Figure 1a.

The basic Latin square of order 3 is in the second quadrant. It is reflected into the first quadrant by mirror image and then both quadrants 1 and 2 are lake-reflected to yield quadrants 3 and 4.

The main thing here is that zeros fill the diagonals of the large square.

Step #3.
An Interim Square

Use the values of Table 1, according to quadrant and Latin element to substitute the values into Figure 1. For example, Using Table 1a, we get:

	0	m^2	$2m^2$	m^2	$3m^2$	$2m^2$	
Q2	$2m^2$	0	m^2	$3m^2$	$2m^2$	m^2	Q1
	m^2	$2m^2$	0	$2m^2$	m^2	$3m^2$	
	$2m^2$	$3m^2$	m^2	$3m^2$	0	0	
	$3m^2$	m^2	$2m^2$	0	$3m^2$	0	
Q3	m^2	$2m^2$	$3m^2$	0	0	$3m^2$	Q4

Figure 2a.

Now you can see that in Figure 2a, the sum of each row, each column and each diagonal is $9m^2$.

Step #4.
The Supplemental Square

Evaluate Figure 2. In this case m=3, so we have:

0	9	18	9	27	18
18	0	9	27	18	9
9	18	0	18	9	27
18	27	9	27	0	0
27	9	18	0	27	0
9	18	27	0	0	27

Figure 3a. Supplemental Square.

The supplemental square can be used with any third-order magic square, providing the following procedure is done.

Step #5. Fully Reflected Magic Square

If your original magic square is:

2	9	4
7	5	3
6	1	8

then place it in quadrant 2 and fully reflect it into the other three quadrant. [Later, place it in another quadrant, fully reflect it and obtain other sixth-order magic squares.]

2	9	4	4	9	2
7	5	3	3	5	7
6	1	8	8	1	6
6	1	8	8	1	6
7	5	3	3	5	7
2	9	4	4	9	2

Figure 4a. Fully reflected magic square of order 3.

Step #6. The Finished Magic Square of order 6.

Simply add the numbers in Figure 3a to the numbers in Figure 4a, in one-to-one correspondence and obtain Figure 5a, the completed magic square.

2	18	22	13	36	20
25	5	12	30	23	16
15	19	8	26	10	33
24	28	17	35	1	6
34	14	21	3	32	7
11	27	31	4	9	29

Figure 5a.

The magic square can be checked to see the magic sum is S=111. Then one can also check to see if there is more. In this case, we have an inlaid magic square of order 3 in the first quadrant which sums 69.

We may now make a different one.

Q	0	1	2	Sum
1	0	$3m^2$	$2m^2$	$5m^2$
2	m^2	0	$3m^2$	$4m^2$
3	$3m^2$	m^2	m^2	$5m^2$
4	$2m^2$	$2m^2$	0	$4m^2$
S	$6m^2$	$6m^2$	$6m^2$	$18m^2$

Table 1e.

Substituting these values into Figure 1a, we get:

m^2	0	$3m^2$	$2m^2$	$3m^2$	0
$3m^2$	m^2	0	$3m^2$	0	$2m^2$
0	$3m^2$	m^2	0	$2m^2$	$3m^2$
m^2	m^2	$3m^2$	$2m^2$	0	$2m^2$
m^2	$3m^2$	m^2	$2m^2$	$2m^2$	0
$3m^2$	m^2	m^2	0	$2m^2$	$2m^2$

Figure 2e.

Evaluating Figure 2e, we obtain:

9	0	27	18	27	0
27	9	0	27	0	18
0	27	9	0	18	27
9	9	27	18	0	18
9	27	9	18	18	0
27	9	9	0	18	18

Figure 3e.

Figure 3e may now be used with any third-order magic square. Comining this with Figure 4a, we achieve:

11	9	31	22	36	2
34	14	3	30	5	25
6	28	17	8	19	33
15	10	35	26	1	24
16	32	12	21	23	7
29	18	13	4	27	20

Figure 5e.

Comparing this to Figure 5a, "1" & "36" are in the same positions, but otherwise it is quite different.

A MAGIC SQUARE OF ORDER TEN

A new table is required.

Q	0	1	2	3	4	Sum
1	$2m^2$	$3m^2$	$3m^2$	$2m^2$	0	$10m^2$
2	0	m^2	m^2	0	$3m^2$	$5m^2$
3	m^2	$2m^2$	$2m^2$	$3m^2$	$2m^2$	$10m^2$
4	$3m^2$	0	0	m^2	m^2	$5m^2$
S	$6m^2$	$6m^2$	$6m^2$	$6m^2$	$6m^2$	$30m^2$

Table 1f.

Table 1f is just one of many possible charts and in each chart the columns headed by 1 to 4 can be rearranged in 24 possible ways.

The basic Latin square is of fifth-order and becomes Figure 1f which is not snown. Of course, Figure 1f, similar to Figure 1a, is fully reflected into the first, third and fourth quadrants. from its initial beginnings in the second quadrant.

Then, the digits are assigned from Table 1f to produce an interim square Figure 2f which is shown below.

0	m^2	m^2	0	$3m^2$	0	$2m^2$	$3m^2$	$3m^2$	$2m^2$
$3m^2$	0	m^2	m^2	0	$2m^2$	$3m^2$	$3m^2$	$2m^2$	0
0	$3m^2$	0	m^2	m^2	$3m^2$	$3m^2$	$2m^2$	0	$2m^2$
m^2	0	$3m^2$	0	m^2	$3m^2$	$2m^2$	0	$2m^2$	$3m^2$
m^2	m^2	0	$3m^2$	0	$2m^2$	0	$2m^2$	$3m^2$	$3m^2$
$2m^2$	$2m^2$	$3m^2$	$2m^2$	m^2	$3m^2$	m^2	m^2	0	0
$2m^2$	$3m^2$	$2m^2$	m^2	$2m^2$	0	$3m^2$	m^2	m^2	0
$3m^2$	$2m^2$	m^2	$2m^2$	$2m^2$	0	0	$3m^2$	m^2	m^2
$2m^2$	m^2	$2m^2$	$2m^2$	$3m^2$	m^2	0	0	$3m^2$	m^2
m^2	$2m^2$	$2m^2$	$3m^2$	$2m^2$	m^2	m^2	0	0	$3m^2$

Figure 2f.

Since the original square in this case is a 5th-order magic square, m=5 and Figure 3f, a supplemental square emrges. This pattern then can be used for any fifth-order square to make a tenth-order one.

Figure 4f is made from the inital fifth-order magic square and then the final tenth-order magic square is made by simply adding together Figures 3f and 4f to produce 5f.

0	25	25	0	75	0	50	75	75	50
75	0	25	25	0	50	75	75	50	0
0	75	0	25	25	75	75	50	0	50
25	0	75	0	25	75	50	0	50	75
25	25	0	75	0	50	0	50	75	75
50	50	75	50	25	75	25	25	0	0
50	75	50	25	50	0	75	25	25	0
75	50	25	50	50	0	0	75	25	25
50	25	50	50	75	25	0	0	75	25
25	50	50	75	50	25	25	0	0	75

Figure 3f.

Supplemental square.

8	15	17	24	1	1	24	17	15	8
22	4	6	13	20	20	13	6	4	22
11	18	25	2	9	9	2	25	18	11
5	7	14	16	23	23	16	14	7	5
19	21	3	10	12	12	10	3	21	19
19	21	3	10	12	12	10	3	21	19
5	7	14	16	23	23	16	14	7	5
11	18	25	2	9	9	2	25	18	11
22	4	6	13	20	20	13	6	4	22
8	15	17	24	1	1	24	17	15	8

Figure 4f.

Fully reflected fifth-order magic square

8	40	42	24	76	1	74	92	90	58
97	4	31	38	20	70	88	81	54	22
11	93	25	27	34	84	77	75	18	61
30	7	89	16	48	98	66	14	57	80
44	46	3	85	12	62	10	53	96	94
69	71	78	60	37	87	35	28	21	19
55	82	64	41	73	23	91	39	32	5
86	68	50	52	59	9	2	100	43	36
72	29	56	63	95	45	13	6	79	47
33	65	67	99	51	26	49	17	15	83

Figure 5f. 10th-order magic square with inlaid 5th-order magic square. S=505 and s=315

MAGIC SQUARES OF EVEN ORDER.

These may be done with identically the same method. All that is required is a table such as:

Q	0	1	2	3	Sum
1	$3m^2$	$3m^2$	m^2	0	$7m^2$
2	m^2	0	$2m^2$	$2m^2$	$5m^2$
3	0	m^2	$3m^2$	$3m^2$	$7m^2$
4	$2m^2$	$2m^2$	0	m^2	$5m^2$
S	$6m^2$	$6m^2$	$6m^2$	$6m^2$	$24m^2$

Table 1g.

One then simply follows the remainder of the six steps as shown on the next page.

0	1	2	3	3	2	1	0
3	0	1	2	2	1	0	3
2	3	0	1	1	0	3	2
1	2	3	0	0	3	2	1
1	2	3	0	0	3	2	1
2	3	0	1	1	0	3	2
3	0	1	2	2	1	0	3
0	1	2	3	3	2	1	0

Figure 1g.

m^2	0	$2m^2$	$2m^2$	0	m^2	$3m^2$	$3m^2$
$2m^2$	m^2	0	$2m^2$	m^2	$3m^2$	$3m^2$	0
$2m^2$	$2m^2$	m^2	0	$3m^2$	$3m^2$	0	m^2
0	$2m^2$	$2m^2$	m^2	$3m^2$	0	m^2	$3m^2$
m^2	$3m^2$	$3m^2$	0	$2m^2$	m^2	0	$2m^2$
$3m^2$	$3m^2$	0	m^2	$2m^2$	$2m^2$	m^2	0
$3m^2$	0	m^2	$3m^2$	0	$2m^2$	$2m^2$	m^2
0	m^2	$3m^2$	$3m^2$	m^2	0	$2m^2$	$2m^2$

Figure 2g.

16	0	32	32	0	16	48	48
32	16	0	32	16	48	48	0
32	32	16	0	48	48	0	16
0	32	32	16	48	0	16	48
16	48	48	0	32	16	0	32
48	48	0	16	32	32	16	0
48	0	16	48	0	32	32	16
0	16	48	48	16	0	32	32

Figure 3g.

1	15	14	4	4	14	15	1
12	6	7	9	9	7	6	12
8	10	11	5	5	11	10	8
13	3	2	16	16	2	3	13
13	3	2	16	16	2	3	13
8	10	11	5	5	11	10	8
12	6	7	9	9	7	6	12
1	15	14	4	4	14	15	1

Figure 4g.

17	15	46	36	4	30	63	49
44	22	7	41	25	55	54	12
40	42	27	5	53	59	10	24
13	35	34	32	64	2	19	61
29	51	50	16	48	18	3	45
56	58	11	21	37	43	26	8
60	6	23	57	9	39	38	28
1	31	62	52	20	14	47	33

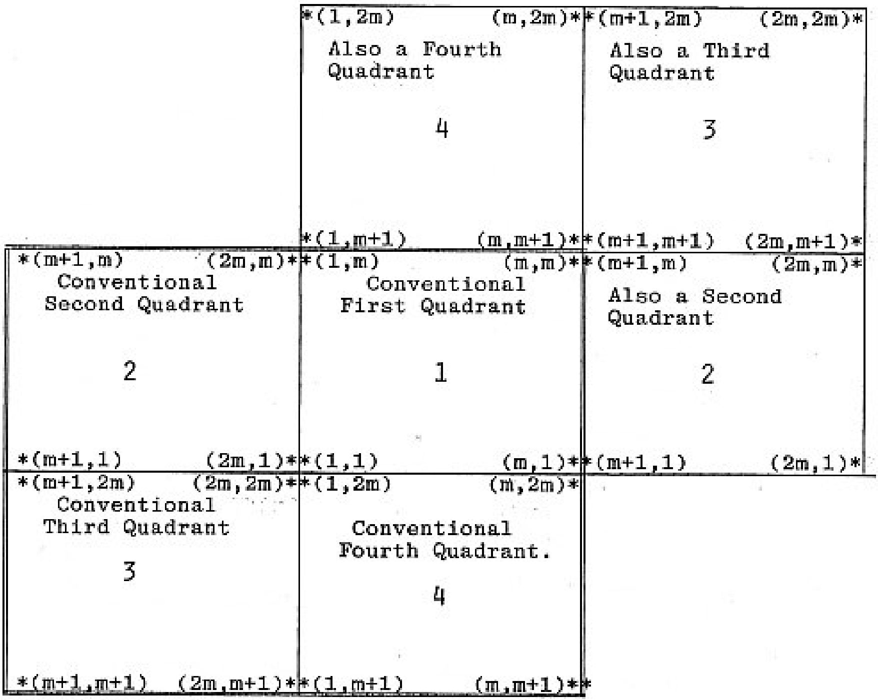
Figure 5g.
Magic Square Order 8.S=260

Magic cubes can also be made by a method similar to this, as is the method used in Chapter 9 of the book Magic Squares to Tesseracts by Computer.

QUADRANTS

A magic square of order 2m is an even-ordered square and can be divided into four regions called quadrants. You may think of quadrants in the conventional sense, except that our axis reference system contains no origin, no negative numbers, and it contains only integers.

In Eulerian space, which is bounded and reentrant, if you go in the x-direction long enough, you get back where you started. The same with the y-direction.



Most people want the numbers running from 1 to 2m which is found in the top right part of this chart. The four corner coordinates are shown for each quadrant.

Interchange of columns in the table

Q	0	1	2	Sum
1	0	$2m^2$	$2m^2$	$4m^2$
2	m^2	$3m^2$	m^2	$5m^2$
3	$3m^2$	m^2	0	$4m^2$
4	$2m^2$	0	$3m^2$	$5m^2$
S	$6m^2$	$6m^2$	$6m^2$	$18m^2$

Table 1.

26	1	33	6	10	35
30	23	7	16	32	3
4	36	20	29	9	13
22	27	2	11	18	31
21	5	25	34	14	12
8	19	24	15	28	17

Magic Square #1.

Q	0	1	2	Sum
1	0	$2m^2$	$2m^2$	$4m^2$
2	m^2	m^2	$3m^2$	$5m^2$
3	$3m^2$	0	m^2	$4m^2$
4	$2m^2$	$3m^2$	0	$5m^2$
S	$6m^2$	$6m^2$	$6m^2$	$18m^2$

Table 2.

26	28	6	15	1	35
3	23	34	7	32	12
31	9	20	29	18	4
22	27	2	11	36	13
21	5	25	16	14	30
8	19	24	33	10	17

Magic Square #2.

Since quadrant #1, lower left, is produced by 0 $2m^2$ $2m^2$ in both tables, it turns out to be the same in both squares.

Since quadrants 1 and 3; and 2 and 4 sum $3m^2$ in the zero column both squares have the same diagonals.

Otherwise, there is a large change.

S=2925

16	87	121	156	152	108	113	289	56	137	127	275	270	71	318	283	249	97
37	74	151	105	112	129	143	99	248	86	261	305	48	274	267	313	155	118
315	23	28	128	142	96	85	120	161	323	282	4	258	304	290	109	104	153
111	296	63	14	82	119	160	150	103	265	69	322	281	244	95	144	134	273
141	94	246	44	81	149	100	110	133	52	272	262	311	162	125	84	256	303
83	124	159	310	21	35	135	140	91	253	302	297	116	102	148	321	286	2
158	145	101	115	294	58	12	89	126	288	251	93	139	132	277	263	64	320
107	117	131	136	92	250	42	76	147	309	157	123	88	254	298	50	279	269
130	138	98	90	122	154	308	25	33	114	106	146	316	284	9	260	300	292
211	219	179	171	203	235	227	187	195	276	268	65	73	41	252	17	57	49
188	198	212	217	173	169	204	238	228	66	319	285	7	11	55	293	36	26
239	226	182	196	213	220	174	170	207	45	8	255	301	51	34	20	307	77
164	205	240	229	183	197	216	221	172	10	59	54	278	264	67	78	43	245
222	175	165	206	243	230	181	191	214	295	29	19	68	324	287	3	13	60
192	215	225	176	163	200	241	231	184	22	312	79	38	1	257	306	53	30
234	185	190	209	223	177	166	201	242	80	39	247	15	61	47	271	266	72
199	236	232	186	193	210	224	180	167	5	18	62	291	31	24	70	317	280
178	168	202	237	233	189	194	208	218	299	46	32	27	314	75	40	6	259

Figure. A magic square of order 18 which sums 2925 in each row, each column and in both diagonals. Inlaid, is a pandiagonal magic square of order 9, which sums 1827.

At one time, pandiagonal magic squares of order nine were thought to be impossible. This one is formed of consecutive numbers 163...243 and embedded in an 18th-order magic square. Magic squares of orders 4k+2, k=1,2,3,... are known to be the most difficult. Here, two difficult squares are combined into one diagram.

\- John R. Hendricks

Books for Sale

Just over 200 pages each, run on 22x28 cm paper, spiral bound, card covers, indexed, more than 100 diagrams.	PRICE IN Canadian Dollars Postpaid	PRICE IN American Dollars Postpaid
Magic Squares to Tesseracts by Computer. Uses modular equations to make magic squares, cubes and tesseracts. Large coverage of geometry involved.	\$32.00 ea.	\$25.00 ea.
Inlaid Magic Squares and Cubes. Employs algebraic digits. The world's first inlaid cubes are shown.	\$32.00 ea.	\$25.00 ea.
BOOKLETS FOR SALE		
36 pages each, run on 22 x 28cm paper, card covers and stapled. These are supplements to Magic Squares to Tesseracts by Computer.		
All Third-Order Magic Tesseracts. This is a catalogue of all 58 possible magic tesseracts of order three.	\$ 8.00 ea.	\$ 6.00 ea.
Perfect n-Dimensional Magic Hypercubes of Order 2^n. Gives the specifications, equations, and BASIC programs for perfect magic hypercubes in 4th and 5th-dimension. Extra geometry is provided.	\$8.00 ea.	\$ 6.00 ea.

First: If you require a set of one of each of the above, then subtract 10% from the total order.

Second: If you require five of any one book, or booklet, then subtract 10% from the total order.

Third: If you reside outside the United States or Canada and you wish airmail, then please add 40% to the total cost.

Please print your name and address clearly. Use, if possible, an international money-order payable to John R. Hendricks, Apt. 308 - 151 St. Andrews St., Victoria, B.C., V8V 2M9, CANADA. Use either currency. Please state clearly what you require.