

MAGIC SQUARE BORDERS

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The idea behind bordering a magic square is to make a larger square out of a smaller one. There are various books that show something on the subject.

4	11	14	5
16	NUCLEUS		1
2	ORDER 2		15
12	6	3	13

Figure 1.

Figure 1 shows that even though a magic square of order two does not exist, it may be bordered. The inner square is called the nucleus.

4	9	2
3	5	7
8	1	6

Figure 2.

12	17	10
11	13	15
16	9	14

Figure 3.

Start with an order 3 magic square, as shown in Figure 2 and add 8 to every number and you will obtain Figure 3. Addition, subtraction, multiplication and division can be done like this and the square will remain magic, as long as it is done to every term. The magic sum is 15 in one square and 29 in the other square.

You cannot just place the numbers 10, 11, 12,..., 24, 25 around Figure 2 because they upset the balance. In advance we also determine the magic sums by the formula, or Equation 1.:

$$S_m = m(1 + m^2)/2 \quad \dots(1)$$

Where. M is the order of the magic square.

$$S_3 = 15 \quad S_4 = 34 \quad S_5 = 65 \quad S_6 = 111$$

What we do is to consider that the bordered square has three sets of numbers: low, medium and high. The medium set is the shown in Figure 3. This means that there are the two sets to be considered as shown below:

Low set:	1	2	3	4	5	6	7	8
High set:	25	24	23	22	21	20	19	18

These are the ones to place around the square and please notice that they have been shown here in pairs which sum 26. One solution is:

3	4	18	21	19
6	12	17	10	20
24	11	13	15	2
25	16	9	14	1
7	22	8	5	23

Figure 4 .

You can see how the high and low numbers are opposites across the square. Notice also that 4 and 18 can be interchanged, providing that you also interchange 22 and 8. In fact, the numbers 4, 18 and 21 can be placed in 6 different ways, as can 6, 24 and 25 giving you a total of 36 variations by simply exchanging border elements.

There are other kinds of magic squares within magic squares. There is what is called the “Inlaid square” which is done by an entirely different process. It uses an expansion band, instead of a border.

Normalizing the Border

Pretend that you have worked all morning and have come up with Figure 5.

8	24	4	23	6
5				21
7				19
25				1
20	2	22	3	18

← Lowest Corner

Figure 5.

You want to know if it is already known. So you consult the catalogue of 5th order borders. Borders have to be listed in some kind of numerical order. The lowest corner number is often put top-left for magic squares (otherwise, it would be bottom left.) Then, then the interchangeable border elements are also arranged in the row and column through the corner in numerical order, This looks like Figure 6.

6	1	19	21	
4				
23				
24				

Figure 6.

You can now look this up in the Table of Order 5 Borders.

TABLE OF ORDER 5 BORDERS

NORMALIZED

1	2	18	21	23	1	2	19	20	23
19				7	18				8
20		#1		6	21		#2		5
22				4	22				4
3	24	8	5	25	3	24	7	6	25
2	4	20	21	18	3	2	20	21	19
7				19	8				18
23		#3		3	22		#4		4
25				1	25				1
8	22	6	5	24	7	24	6	5	23
3	4	18	21	19	4	1	19	21	20
6				20	10				18
24		#5		2	23		#6		3
25				1	24				2
7	22	8	5	23	6	25	7	5	22
4	2	18	21	20	5	1	18	22	19
7				19	6				20
23		#7		3	23		#8		3
25				1	24				2
6	24	8	5	22	7	25	6	4	21

