CALCULUS
LAB - ZOOMING IN

You learned in Algebra that the slope of a non-vertical straight line is $\frac{\Delta y}{\Delta x}$ or $\frac{y_2 - y_1}{x_2 - x_1}$.

Most functions we see in Calculus have the property that if we pick a point on the graph of the function and zoom in, we will see a straight line. When a function has this property, we say that it has **local linearity**.

1. Graph the function $f(x) = x^4 - 10x^2 + 3x$ in a friendly window. Zoom in on the point (3, 0) until the graph looks like a straight line. Pick a point on the curve near the curve near the point (3, 0) and estimate the coordinates of the point. Store the x-coordinate in a and the y-coordinate in b. Calculate the slope of the line through those two points. The number computed is an approximated to the **slope of the function** $f(x) = x^4 - 10x^2 + 3x$ at the point (3, 0). This slope is also called the **derivative of f at $x = 3$** and is denoted $f'(3)$.

2. Use zooming to estimate the slope of the following functions at the specified points.

a) $f(x) = x^4 - 6x^2$ at $(1, -5)$ Slope = _____

b) $f(x) = \cos x$ at $\left(\frac{\pi}{2}, 0\right)$ Slope = _____

c) $f(x) = (x-1)^{1/3}$ at $(2, 1)$ Slope = _____

So far in this lab you have used the graph of a function to estimate the value of its derivative at a specified point. Sometimes, however, a function does not have a slope at a point and therefore has no derivative there.

3. Graph $f(x) = (x-1)^{1/3}$ again. This time, zoom in on (1, 0). Describe what you see. Do you think that $f'(1)$ exists? Support you answer with an appropriate sketch.

4. Graph $f(x) = |x+1|$. By looking at the graph and zooming in on points you select, decide at which points the function f has a derivative and at which points it does not. Support your answer with an appropriate sketch.

5. The derivative of f at a point $x = a$ is defined analytically by the formula

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

Explain in your own words how calculating the slope of a function at the point $(a, f(a))$ by repeated zooming is related to the computation of the derivative $f'(a)$ by this definition.

Checking the accuracy of your numerical derivatives

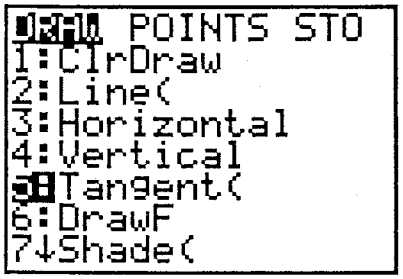
Check the accuracy of your slope predictions using graphing technology. Using the TI-83 Plus calculator:

i) Enter the function into Y1 of the equation editor and press ZOOM 8 to graph the function using an integer scale.

ii) Move the cursor to the origin and press ENTER

iii) Using the TRACE key move the cursor to (3, 0)

iv) Draw the tangent line by pressing 2nd PRGM 5, and press ENTER



v) The equation of the tangent line is given in the form y = mx + b. Compare the slope in the equation to your prediction and adjust the value in your table if necessary.

or

Go to your home screen and calculate the numerical derivative of your function $f(x)$.

Type MATH 8 ... **nDeriv(Y1, X, 3)**

to find the numerical derivative of $f(x)$ at x = 3.