A Brief Review of Functions

A **function** is a rule that assigns to each element in a set X (the **domain**) exactly one element in a set Y (the **range**).

You can think of functions as "little black boxes." You put something into a function, and it spits something back out.

x ☞ f	☛	f(x)

The important distinguishing feature about functions is for every input there is one and only one output.

We can call a function anything we want, usually we call a function *f*. (Other popular names are *h* and *k*.) This *name* just helps us distinguish between more than one function. We usually work with functions that are defined on the real numbers, that is, our domain is the set of real numbers (or some subset of the real numbers).

We designate our inputs as *x*, also known as the independent variable. Then we designate our output as *f(x)*, which we read "f of x". It does **NOT** mean multiply f times x.

When we can write our function "rule" as an equation, we can write, for example,

$$f(x) = x^2 + 3x - 1$$

Then, we can evaluate our function at different numbers. That is, we can throw different numbers into the "box" and see what comes out. Sometimes it's helpful to think of your function like this:

$$f(\ ) = (\ )^2 + 3(\ ) - 1$$

Then, if we want to evaluate our function at 2, we just stick a 2 into all the blanks:

$$f(2) = (2)^2 + 3(2) - 1 = 4 + 6 - 1 = 9$$

so we would say *f(2) = 9.* Similarly we can find *f(z):*

$$f(z) = (z)^2 + 3(z) - 1 = z^2 + 3z - 1$$

or even *f(x+h)* (which we will be doing a lot of next semester in Calculus).

$$\begin{aligned} f(x+h) &= (x+h)^2 + 3(x+h) - 1 \\ &= (x^2 + 2hx + h^2) + (3x + 3h) - 1 \\ &= x^2 + 2hx + h^2 + 3x + 3h - 1 \end{aligned}$$

Linear Functions

A linear function is a function with an equation of the form y = mx + b. The graph of the function is a line with slope *m*

$$m = \frac{RISE}{RUN} = \frac{y_2 - y_1}{x_2 - x_1}$$ where (x_1, y_1) and (x_2, y_2) are points on the line

and y-intercept (0, *b*).

The **identity function**, i(x) = x and **constant function**, f(x) = c where c is any real number are special cases of a linear function.

Assignment: Pg. 12 #'s 7, 8

Absolute Value Function

The absolute value function y = $|x|$ can be defined as a **piecewise function**:

$$f(x) = \begin{cases} x & x >= 0 \\ -x & x < 0 \end{cases}$$

See example 2 page 9.

Assignment: Pg. 11 - 12 #'s 4c, 10

Quadratic Function

A quadratic function is a polynomial function of degree 2 with an equation that can be written in the form:

$f(x) = ax^2 + bx + c$ where *a*, *b*, and *c* are constants and $a \neq 0$

The graph of every quadratic function is a curve called a parabola.

Assignment: Pg. 10 - 11 #'s 1, 4d

Cubic Function

A cubic function is a polynomial function of degree 3. Two examples of cubic functions are $y = x^3$ and $y = x^3 - 7x$.

Assignment: Sketch each cubic function above (use a separate grid for each) and then determine for each:

a) the domain and range

b) any real zeros (to the nearest tenth)

c) the y-intercept

d) the coordinates of any relative maximums and minimums (to the nearest tenth)

Polynomial Functions

A polynomial function of degree n is a function whose equation can be written in the form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0$$

where *n* is a whole number and $a_n, a_{n-1}, \dots, a_1, a_0$ are real numbers. The coefficient of the highest power of x is called the leading coefficient and the highest power is called the degree of the polynomial.

Linear, quadratic and cubic functions are all examples of polynomial functions. Polynomial functions of degree four and five are called **quartic** and **quintic** functions respectively.

Power Function

Functions whose equations have the form:

$f(x) = cx^a$ where c is a (constant) real number not equal zero and $a > 0$

are called power functions.

Assignment: Pg. 12 # 9

Reciprocal Function

A reciprocal function is a function in the form $\frac{1}{f(x)}$ where f(x) $\neq$ 0. The simplest reciprocal function, $y = \frac{1}{x}$ has a graph (see page 7) which is a hyperbola whose asymptotes are the coordinate axis.

Assignment: Pg. 10 - 11 #'s 2, 4b

Radical Function

A function such as $y = \sqrt[3]{x} + 1$, which has a variable in a radicand, is called a radical function. A radical function containing only a square root is called a **square-root function** (see page 7.)

Assignment: Pg. 10 - 11 #'s 3, 4(a, e), 5

Rational Function

A rational function is a function of the form $f(x) = \frac{g(x)}{h(x)}$, where g(x) and h(x) are polynomial functions and h(x) $\neq$ 0.

Assignment: Sketch the graph of the rational function $y = \frac{3}{x-2}$ in your notebook.

a) State the domain and range.

b) Determine the equation of any horizontal or vertical asymptotes.

Composition of functions

When we compose two functions *f(x)* and *g(x)* we use the outputs of one function as the inputs of the other. When we compose two functions the result is a new function. The **composite function**

$f \circ g$ **or** $(f \circ g)(x) = f(g(x))$ . . . we read this a "*f* of *g* of *x*"

takes the original inputs *x* and substitutes them into the inside function, here *g(x)*. The output generated by this substitution is then used as the input of the outer function, *f(x)*.

As an example, consider if $f(x) = 4x^2$ and $g(x) = 3x - 2$:

$$f(g(x)) = f(3x - 2) = 4(3x - 2)^2$$
$$= 4(9x^2 - 12x + 4)$$
$$= 36x^2 - 48x + 16$$

In a similar way we can find

$g \circ f$ **or** $(g \circ f)(x) = g(f(x)) = g(4x^2) = 3(4x^2) - 2$
$$= 12x^2 - 2$$

The Inverse of a Function

The inverse of a function is the set of ordered pairs obtained by interchanging the members of each ordered pair of the function. As a simple example, if we have a function:

$f(x) = \{ (1, 2), (3, 4), (5, 6) \}$

its inverse (which, in this case, also turns out to be a function that we will call $f^{-1}(x)$) is:

$f^{-1}(x) = \{ (2, 1), (4, 3), (6, 5) \}$

Functions are said to be inverses of each other if

$f \circ f^{-1} = f^{-1} \circ f = x$

and/or if the graph of *$f^{-1}(x)$* is a reflection of the graph of *$f(x)$* in the line *$y = x$*.

If f(x) = 3x - 2, we can find its inverse as follows:

f(x) = 3x - 2

y = 3x - 2 . . . replace f(x) with y
x = 3y - 2 . . . interchange the x and y variables
3y = x + 2 . . . solve for y

$y = \frac{x+2}{3}$ . . . and finally, replace y with $f^{-1}(x)$

$f^{-1}(x) = \frac{x+2}{3}$

To test if the above are in fact inverses of each other we find:

$f \circ f^{-1} = f(f^{-1}(x)) = f(\frac{x+2}{3}) = 3(\frac{x+2}{3}) - 2 = x + 2 - 2 = x$

$f^{-1} \circ f = f^{-1}(f(x)) = f^{-1}(3x - 2) = \frac{(3x-2)+2}{3} = \frac{3x}{3} = x$

We will consider the **Reflection Property of Inverses** later in Chapter 1.