PROBLEM 1

Continuous growth rate versus annual growth rate

> ***Definition:*** The natural logarithm of x, written $\ln x$, is defined to be the inverse function of e^x.
>
> $$\ln x = \log_e x = c \quad \Rightarrow \quad x = e^c$$

***Note*:** When giving calculator approximations, express answers accurate to 5 decimal places.

a) Use your calculator to evaluate:

i) $\ln 1$ ii) $\ln e$ iii) $\ln 10$ iv) $\ln e^3$

b) Use your calculator to evaluate:

i) e^0 ii) e^1 iii) e^{-1} iv) $e^{\ln 5}$

c) Use the results from parts a) and b) to derive a formula for $\ln e^x$ and $e^{\ln x}$.

d) Solve the following equation for k by taking the natural logarithm of both sides.

$$1.035 = e^k$$

e) A recent survey showed that the population, P, of Victoria is growing at an annual rate of 1.1%. Let P_0 represent the population on January 1, 2001 and let t represent the time, in years, since this date.

i) Express P as a function of t in the form $P = P_0 a^t$, where a is an appropriate constant.

ii) Since population can be considered to grow continuously, express the same function P as an exponential function of t using base e, in the form $P = P_0 e^{kt}$, where k is an appropriate constant.

iii) k is called the continuous growth rate. Compare this growth rate to the annual growth rate of 1.1%. Which is larger, and by how much? How significant is this difference? For example, if $P_0 = 325\,000$, determine the value of P in 10 years using each formula.

iv) The value of k found in part ii) is quite close to the annual growth rate of 1.1%. Therefore, it is often said that the model function $P = P_0 e^{rt}$ gives a good approximation for a population that is increasing annually by the rate of r%. Use this formula with $r = 1.1\%$ and $P_0 = 325\,000$ to determine the value of P in 10 years. Compare this answer to those calculated in parts i) and ii). How significant is the difference?

f) If a bank account earns interest at a rate of 6% per year compounded continuously, determine the *effective annual growth rate.*

Note: The *effective annual growth rate* is the percent increase of the initial amount over one year, assuming the amount was compounded yearly.